

A New Look at a Common Active Filter Circuit (or how a filter with zero damping is also an oscillator)

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Introduction

The 2 pole Sallen and Key active filter is very well known and can be adjusted to give different stop band responses. It also, however can be used as an oscillator and redrawing the circuit, it will be seen to be nothing but the familiar Wein Bridge oscillator.

Sallen and Key Filter

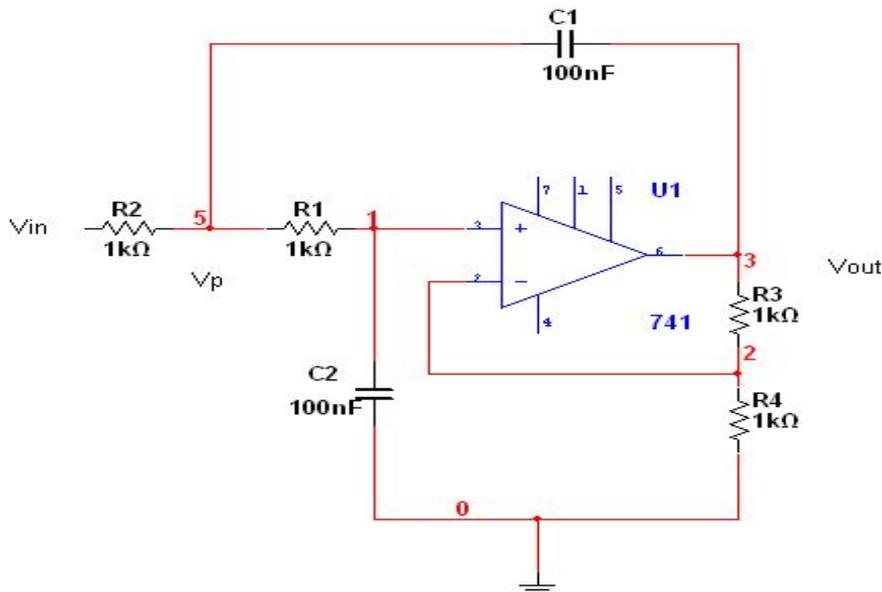


FIG 1: Sallen and Key Low Pass Filter

The basic low pass version is shown above

Analysis:

We can apply Millmans theorem to V_p above, hence

$$V_p(s) = \frac{\left[\left(\frac{V_{in}}{R} \right) + \left(\frac{V_{out}}{K \cdot R} \right) + (V_{out} \cdot S \cdot C) \right]}{\left(\frac{1}{R} \right) + \left(\frac{1}{R} \right) + S \cdot C} \dots\dots\dots (1)$$

Where R and C are equal and $K = 1 + R_3/R_4$ the gain of the amplifier.

We can also apply Millmans to node 1 above:

$$V_{out}(S) := \frac{\left(\frac{V_p(S)}{R}\right) \cdot K}{\left(\frac{1}{R}\right) + S \cdot C}$$

If the above is re-arranged to give V_p , then

$$V_p(S) := \frac{V_{out}(S) \cdot (S \cdot C \cdot R + 1)}{K} \dots\dots\dots (2)$$

We can now equate (1) and (2), apply a little algebra and substitute $T = RC$, to give the voltage transfer ratio V_{out}/V_{in} , as a function of S, as follows :

$$\frac{V_{out}(S)}{V_{in}(S)} := \frac{K}{S^2 T^2 + S(3T - TK) + 1}$$

Also if we use the fact that $\omega = 1/T$, then dividing through by T^2 , we get

$$\frac{V_{out}(S)}{V_{in}(S)} := \frac{K \cdot \omega^2}{S^2 + S(3 - K) \cdot \omega + \omega^2}$$

This is immediately recognisable as the standard 2nd order system, as follows:

$$TF(S) := \frac{K \cdot \omega^2}{S^2 + 2 \cdot \xi \cdot \omega \cdot S + \omega^2}$$

Hence comparing the two expressions,

$$\xi := \frac{(3 - K)}{2}$$

Therefore by adjusting the gain of the amplifier (K), we can choose any damping factor we please and therefore any filter response. For example a Butterworth response, $\zeta = 0.7071$ giving a value

for K as 1.59. Hence $R3/R4 = 0.59$

Of course if we make $K = 3$, then $\zeta = \text{zero}$ and there is then no S term in the denominator. The transfer ratio then becomes:

$$\frac{V_{out}(S)}{V_{in}(S)} := \frac{3 \cdot \omega^2}{S^2 + \omega^2}$$

Taking inverse Laplace transforms, this becomes:

$$V_{out}(t) := \frac{3 \cdot \sin(\omega \cdot t)}{\omega}$$

And we have a continuous sine wave with angular frequency $\omega = 1/CR$, that is, an oscillator.

If we now redraw the circuit, with the input ($R2$ end) connected to ground, then we get the following:

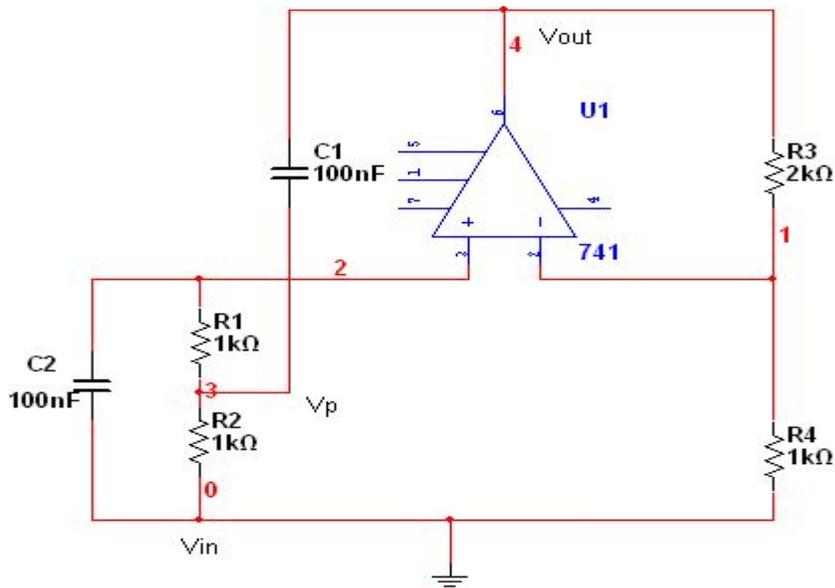


FIG 2: Sallen Key Circuit with input grounded

Now compare this to the familiar Wein Bridge oscillator Fig 3.

Very similar isn't it. The only difference being that the parallel RC circuit is tapped at V_p , and $R1$ is effectively in series with $C1$.

The behaviour of the circuit is identical and the gain of the Wein Bridge is also set equal to 3.

In the Wein Bridge, the phase shift of the series RC circuit is designed to be zero at the frequency of oscillation. Likewise the phase shift of the parallel RC circuit.

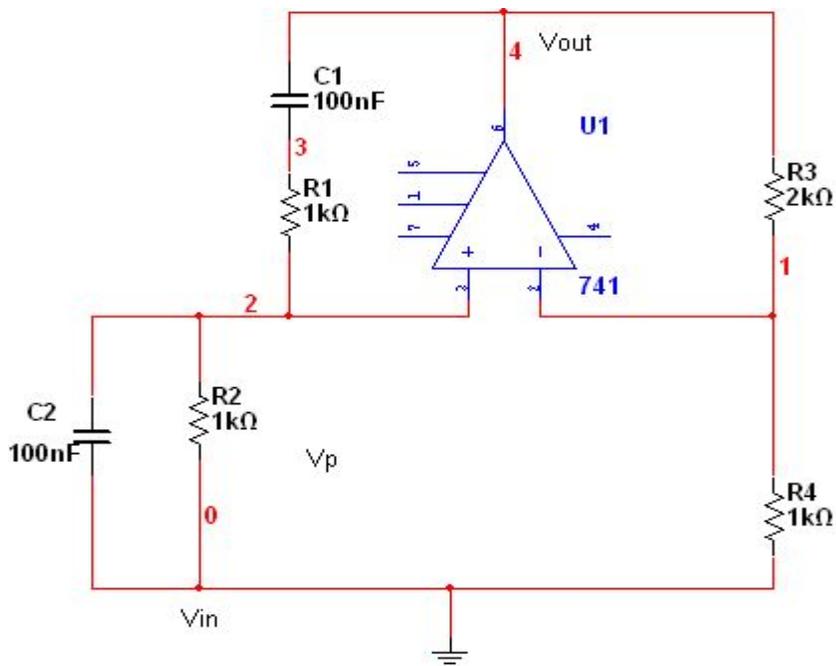


FIG 3: Wein Bridge Circuit

The four arms of the bridge: C1 - R1; C2 - R2; R3; R4

Note as in the Wein Bridge, the gain K needs to be slightly higher than three to kick the circuit into oscillation. Once oscillating K needs to be reduced to three to prevent clipping. This is normally accomplished by a voltage dependent resistor (or filament bulb) as a part of R4.