DERIVATION

Spec F_0 & R_T & K

$$R_T = Z_0 = \sqrt{Z_1 Z_2} \qquad \omega_0 = 2\pi f_0$$

$$X_{L_1} = \omega_0 L_1 \qquad X_{C_1} = \frac{1}{\omega_0 C_1}$$

$$X_{L_2} = \omega_0 L_2 \qquad X_{C_2} = \frac{1}{\omega_0 C_2}$$

Define $L_1 = K L_2$ $\quad$ $C_1 = K C_2$

$$Z_1 = \frac{X_{L_1} . X_{C_1}}{X_{L_1} + X_{C_1}} = \frac{L_1/C_1}{\omega_0 L_1 + \frac{1}{\omega_0 C_1}} = \frac{L_1/\cancel{C_1}(\omega_0 \cancel{C_1})}{\omega_0^2 L_1 C_1 + 1}$$

$$Z_1 = \frac{\omega_0 L_1}{\omega_0^2 L_1 C_1 + 1}$$ ~~But~~

$$\cancel{Z_1} = \cancel{\frac{\omega_0 L_1}{\omega_0^2 L_1 C_1}} =$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$ $\therefore$ $\omega_0^2 = \frac{1}{L_1 C_1}$

$$\therefore \quad Z_1 = \frac{\omega_0 L_1}{1 + 1} = \frac{\omega_0 L_1}{2}$$

$$Z_2 = \omega_0 L_2 + \frac{1}{\omega_0 C_2}$$

Now $Z_1 Z_2 = R_T^2 = \frac{\omega_0 L_1}{2} \times \omega_0 L_2 + \frac{1}{\omega_0 C_2}$

$$= \frac{\omega_0^2 L_1 L_2}{2} + \frac{\cancel{\omega_0} L_1}{\cancel{\omega_0} C_2}$$

$$\therefore R_T^2 = \frac{\omega_0^2 L_1 L_2}{2} + \frac{L_1}{C_2}$$

$$L_2 = \frac{L_1}{K} \qquad \therefore R_T^2 = \frac{\omega_0^2 L_1 L_1}{2k} + \frac{L_1}{C_2}$$

$$= \frac{\omega_0^2 L_1^2 C_2 + 2k L_1}{2k C_2}$$

$$C_2 = \frac{C_1}{K}$$

$$= \frac{\omega_0^2 L_1^2 C_1/k + 2k L_1}{2k C_2}$$

$$\omega_0^2 = \frac{1}{L_1 C_1}$$

$$\therefore R_T^2 = \frac{\frac{1}{\cancel{L_1}\cancel{C_1}} \cdot \frac{L_1^{\cancel{2}\,1} \cdot \cancel{C_1}}{k} + 2k L_1}{2k C_2}$$

$$= \frac{L_1/k + 2k L_1}{2k C_2}$$

$$= \frac{L_1 + 2k^2 L_1}{2k^2 C_2}$$

but $2k^2 L_1 > L_1$

$$\therefore R_T^2 = \frac{\cancel{2k^2} L_1}{\cancel{2k^2} C_2} = \frac{L_1}{C_2}$$

$$\therefore R_T = \sqrt{\frac{L_1}{C_2}}.$$

This is the iterative image resistance for the lattice.

$$R_T^2 = \frac{L_1}{C_2}$$

$$\therefore L_1 = C_2 R_T^2 \qquad \omega_0^2 = \frac{1}{L_2 C_2}$$

$$\therefore L_1 = \frac{R_T^2}{\omega_0^2 L_2} \qquad \text{but} \quad L_2 = \frac{L_1}{K}$$

$$\therefore L_1 = \frac{R_T^2}{\omega_0^2 \frac{L_1}{K}} \qquad \therefore L_1^2 = \frac{K R_T^2}{\omega_0^2}$$

$$\therefore L_1 = \sqrt{\frac{K R_T^2}{\omega_0^2}}$$

$$L_2 = \frac{L_1}{K}$$

$$C_2 = \frac{L_1}{R_T^2}$$

$$C_1 = \frac{1}{L_1 \omega_0^2}$$

So	R_T	Derived	from	Z_0
	L_1	"	"	R_T & ω_0
	L_2	"	"	L_1
	C_2	"	"	L_1 & R_T
	C_1	"	"	L_1 & ω_0